



A new design method for longitudinally stiffened plates

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Abstract

Longitudinally stiffened steel plates are becoming increasingly important in bridge design as the size and span of new bridges increases. This paper presents a new method for the design of longitudinally stiffened plates, with or without transverse stiffeners. The method considers explicitly the three effects of plate bending, plate torsion and flexural buckling from the member buckling curve. It combines these contributions to give the buckling resistance of the stiffened plate including the long yield plateau arising from plate behaviour. Explicit consideration and combination of the three effects enables the designer to optimise a structure because the relative importance of each effect is clear. Also, the buckling length of the stiffeners is found by the method so the mode shape is clear to the designer and the spacing of diaphragms can be optimised. The method is suitable for use with effective widths to allow for plate slenderness. The method avoids a limitation of AASHTO LRFD which recognises only the specified minimum stiffness of longitudinal stiffeners when designing plates stiffened longitudinally and transversely. The method of combination of the three effects avoids the anomalies that occur for certain geometries in Eurocode 3 Part 1.5 which uses an interpolation between the buckling curve for unstiffened plates and a buckling curve for compression members. The method involves no iterations and is suitable for application in a design office by spreadsheet or pencil and paper calculations.

1. Why propose a new method

1.1 Introduction

Longitudinally stiffened steel plates are becoming increasingly important in bridge design as the size and span of new bridges increases. Now that most large bridges are part of design-build-finance contracts, it is especially important that the design process be as simple as possible and can be completed rapidly and accurately for the bid design phase because this is the phase that determines the price of the project. This paper is the result of designing several longitudinally stiffened bridge structures ranging in size from footbridges to the 400 metre high towers of the proposed Messina Straights suspension bridge between Italy and Sicily. It proposes a design method that avoids some of the shortcomings or anomalies that engineers find in current design

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codes. The paper first explains why a new method is desirable for designers, then states the proposed method and finally gives the origins of the terms in the proposed equations. Design rules for longitudinally stiffened plates have been available in numerous design codes for many years. The codes that are used most frequently are AASHTO LRFD and the Eurocode, EN 1993-1-5, so this paper considers these two.

In North America, the majority of stiffened plates have only one or two longitudinal stiffeners and very few transverse stiffeners, so a method is needed that gives a reasonably economic design for these stiffened plates. These plates may have resistances significantly higher than predicted by design as simple compression members, especially when the plate thickness is great. This higher strength should not be ignored, but it is complicated to predict.

Where the plates are very wide, they often require several longitudinal stiffeners. For these to be efficient, the plate needs transverse stiffeners that are stiff enough to provide node points for the longitudinal stiffeners to behave as simple compression members. The resistance can be calculated in the same way as a compression member, perhaps with some additional bending applied to account for the curvature induced by the global curvature of the member.

For wide plates with many longitudinal stiffeners, the resistance of the stiffeners is essentially like a continuous compression member because the plate effects are insignificant in a wide plate. For narrower plates, the plate effects become more significant because they add to the resistance of the compression member. Ideally, a design method should reflect this behaviour by simply adding plate effects to the compression member resistance.

1.2 Issues with the existing AASHTO LRFD formulae

The major section in AASHTO LRFD is 6.11.8.2 – Flexural Resistance of Box Flanges in Compression. The resistance formulae use a slenderness derived from (1) the buckling factor ‘k’, as in the general elastic critical buckling formula

$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)\left(\frac{b}{t}\right)^2} \quad (1)$$

where ‘k’ is from Timoshenko’s formulae for elastic critical buckling of longitudinally stiffened plates and (2) the b/t ratio of the plate between the stiffeners or between the web and the adjacent stiffener.

The formulae for plates with one stiffener and with two stiffeners include the inertia of the longitudinal stiffener, but the ‘k’ factor is limited to a maximum of 4. This means that where the plate is relatively thin, the calculated resistance is limited by the buckling of plate between the stiffeners. The greater strength from the effective width approach to plate resistance is not available. Now that the concept of effective widths of plates in compression is recognized as the best model for Ultimate Limit State, it can be seen that the elastic critical buckling of the plate between the stiffeners should not limit the resistance calculated at the Ultimate Limit State.

The formulae for ‘k’ assume that there are no transverse stiffeners to limit the effective length in this buckling mode. It would be helpful to designers to be given a formula to calculate the

effective length of this mode, because many boxes do have transverse stiffeners as part of diaphragms and these could be used to improve the efficiency of the design.

The formulae for plates with more than two stiffeners are in section 6.11.11.2 – Longitudinal Compression-Flange Stiffeners. As in section 6.11.8.2, the slenderness is derived from the elastic critical buckling of the stiffened plate and written in terms a buckling factor ‘k’ and the b/t ratio of the plate between stiffeners . The formula for ‘k’ does not explicitly include the inertia of the longitudinal stiffener. Instead, there is a requirement that the longitudinal stiffeners should satisfy a certain minimum stiffness requirement. This may give a reasonably proportioned stiffened plate, but it reduces the freedom to produce the most desirable structure. The longitudinal stiffeners in a plate with more than three stiffeners behave like compression members spanning between transverse stiffeners and the key property of a compression member is the inertia. If the inertia is not a variable in the resistance formula, the formula cannot respond to the key design parameter. Also, the formula limits the ‘k’ factor to a maximum of 4, so the benefit of effective width approach to plate buckling is not available.

1.3 Issues with the existing Eurocode formulae

The Eurocode (EN1993-1-5) accounts for the plate effects by interpolating between the resistance predicted for a simple compression member (‘column type resistance’) and the resistance predicted for a plate. The interpolation is made because the buckling curve for any particular stiffened plate is not defined. The only buckling curves that are defined are the compression member buckling curve, which has a short plateau length, and the unstiffened plate buckling curve, which has a long plateau length. It forces the designer to make two sets of strength calculations. One of these is relatively simple (column type resistance) and the other is inconvenient for design purposes (plate type resistance), needing either selection of formulae or software.

The interpolation between the compression member buckling curve and the plate buckling curve can produce illogical results in the region of slenderness where the elastic critical buckling of the compression member is higher than the elastic critical buckling of the plate.

2. The proposed method

2.1 Benefits of the proposed method

Designers nowadays are under serious time pressure, so any simplification of design process or clarification of design logic helps in producing efficient designs.

Benefits of the proposed method include:

1. There is only one sequence of calculations to complete which covers both (a) stiffened plates without transverse stiffeners and (b) stiffened plates with transverse stiffeners.
2. The designer can see what the effective length is without transverse stiffeners, enabling the best choice of spacing of diaphragms to be chosen.
3. The plate effects are added to the pinned compression member behaviour, so it is easy to see if the plate effects are worth including or not.
4. The method does not use an interaction formula to select the appropriate amount of plate effects.

2.2 Compression resistance of stiffeners plus plate

The compression resistance of each individual stiffener (including the associated effective width of flange), P_{nS} , is calculated as the resistance of a compression member in flexural buckling restrained by the effects of the plate spanning between the webs. This is calculated as the sum of (a) the flexural buckling resistance of the stiffener restrained by the transverse bending of the plate, P_{nFB} , plus (b) the buckling resistance caused by the torsional stiffness of the plate, P_{nTOR} . Therefore

$$P_{nS} = P_{nFB} + P_{nTOR} \leq P_{yield} \quad (2)$$

2.2.1 Find the buckling effective length

The appropriate buckling length L is taken as the lesser of (a) the spacing of stiff transverse stiffeners and (b) the length for minimum resistance in the absence of transverse stiffeners, given by

$$L = \left(\frac{EI\pi^4}{k_{PB}} \right)^{0.25} \quad (3)$$

where k_{PB} is the transverse bending stiffness of the plate per unit length:

$$k_{PB} = \frac{\pi^4 EI_p}{(n+1)b^3}$$

and I_p is the effective inertia of a unit width of plate, $I_p = \frac{t^3}{12(1-\nu^2)}$

2.2.2 Calculate the flexural buckling resistance, P_{nFB}

The flexural buckling resistance, P_{nFB} , is found from the appropriate flexural buckling curve using the elastic critical buckling load of the longitudinal stiffener restrained by the transverse bending stiffness of the plate, P_{cr} . Expressing this in the simplest and slightly conservative form

$$P_{cr} = EI \frac{\pi^2}{L^2} + k_{PB} \frac{L^2}{\pi^2} \quad (4)$$

where I is the inertia of the longitudinal stiffener plus associated area of plate, and L is the effective buckling length.

Therefore, in AASHTO LRFD 6.9.4.1, P_e is replaced by P_{cr} above, and in EN 1993-1-1 #6.3, the value of N_{cr} would equal P_{cr} above.

2.2.3 Calculate the buckling resistance due to torsion, P_{nTOR}

The flexural buckling resistance, P_{nFB} , cannot have the long plateau of compression resistance that plates are known to exhibit, because flexural buckling curves have either no plateau (in North America) or a plateau much shorter than for plates (in Europe). In the proposed method, the plateau is provided by the buckling resistance caused by the torsional stiffness of the plate, P_{nTOR} .

$$P_{nTOR} = k_{Jf} \times P_{crT} \quad (5)$$

where k_{Jf} is a factor derived from calibration of the method against test results and finite element analyses, and

$$P_{crT} = k_J \frac{Gt^3}{b}$$

where k_J is a factor that varies with the number of stiffeners across the width of the flange.

2.2.4 Calculate the total resistance of the stiffener, P_{nS}

P_{nS} is the sum of (a) the flexural buckling resistance of the stiffener restrained by the transverse bending of the plate, P_{nFB} , plus (b) the buckling resistance caused by the torsional stiffness of the plate, P_{nTOR} , as shown in equation (2)

$$P_{nS} = P_{nFB} + P_{nTOR} \leq P_{yield}$$

2.3 Resistance of restrained widths of plate

Along the line of the webs, the flange plate is restrained from out of plane buckling. At ultimate resistance of the member, this area of plate sustains a higher stress than the stiffeners because of the restraint. Finite element analysis of stiffened flanges shows that the maximum resistance of the flange occurs before these restrained areas reach yield stress, but the stress is higher than the average stress in the stiffeners. Therefore, the resistance of the restrained areas, P_{nS} , should be limited to less than yield in most cases. A reasonable approximation appears to be the mean value of yield stress and stiffener resistance stress.

2.4 Resistance of an entire flange

The resistance of a whole flange subject to uniform compression is calculated as the sum of the resistances of (a) each individual stiffener (including the associated effective width of flange), P_{nS} , plus (b) each effective width of plate attached directly to each web, P_{nR} . Therefore

$$P_{n_flange} = \sum P_{nS} + \sum P_{nR} \quad (6)$$

3. Plate effects

3.1 General

The plate behaviour is dominated by:

- (1) Out-of-plane bending stiffness spanning between the webs
- (2) Torsional stiffness of the plate connecting the stiffener to the adjacent webs

There may also be membrane action between the webs but this effect is usually very small compared with the torsional stiffness and the transverse bending stiffness.

The proposed method accounts for the out-of-plane bending stiffness in the formulae for P_{cr} and for L above. It accounts for torsional stiffness by the term P_{nTOR} above.

3.2 Torsional stiffness and plasticity

The contribution of torsional stiffness to the stability of elements stressed to yield was identified by researchers into plastic behaviour of steel structures in Britain in the 1950s. The importance of the torsional stiffness in the lateral torsional stability of I-beams at their plastic resistance is described in Chapter 12 of Baker, Horne and Heyman (1956). Horne (1964) used this phenomenon in his definition of the stable length of I-beams carrying compression and bending. This is the origin of the formula in Annex BB of Eurocode 3 Part 1.1. It was shown that as the flange force approached the yield, the lateral bending stiffness of the flange (the warping stiffness) is severely reduced because of yielding. The torsional stiffness is reduced, but not as much as the lateral bending stiffness because of the yield criterion.

When applied to a plate in compression, there is similar behavior. As the longitudinal force in the plate approaches the yield force, the ability to resist transverse bending and torsion approach zero as the steel approaches the yield criterion. However, at longitudinal stresses just below yield stress, the available torsional stiffness and resistance are more significant than the available transverse bending stiffness and resistance because of the difference in the effects of shear stresses and direct stresses in the yield criterion.

3.3 Torsional stiffness in buckling of a perfect plate at yield stress

The elastic critical buckling stress of an isotropic flat plate, infinitely long and with simply supported edges is:

$$\sigma_{cr} = \frac{4\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (7)$$

The factor 4 comprises 1.3 from longitudinal bending stiffness, 1.3 from transverse bending stiffness and 1.4 from torsional stiffness.

If the longitudinal stress were increased so close to yield that the longitudinal and lateral bending stiffness were reduced to nearly zero, leaving only the torsional stiffness, then the buckling stress would be:

$$\sigma_{cr} = \frac{1.4\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{b}\right)^2 \quad (8)$$

This buckling is occurring when the applied stress is approximately equal to the yield stress, so the b/t of the plate for the case of yield = buckling is found from:

$$\left(\frac{b}{t}\right)^2 = \frac{1.4\pi^2 E}{12(1-\nu^2)\sigma_{cr}} = \frac{1.4\pi^2 E}{12(1-\nu^2)F_y} = \frac{1.4\pi^2 \times 4E}{48(1-\nu^2)F_y} = 0.316 \left(\frac{4E}{F_y}\right) \quad (9)$$

Therefore the limiting b/t at yield is:

$$\frac{b}{t} = 0.562 \sqrt{\frac{4E}{F_y}} \quad (10)$$

giving b/t = 27.1 for a plate of 345 MPa assuming E = 200,000 MPa.

This is very close to AASHTO LRFD eqn 6.11.8.2.2-9, which can be written as:

$$\frac{b}{t} = 0.57 \sqrt{\frac{4E}{F_y}} \quad (11)$$

For comparison, Eurocode 3 Part 1.5 Section 4.4(2) gives a plateau up to b/t = 31.5 for 345MPa yield, assuming the Eurocode value of E = 210000MPa.

This supports the thesis that the torsional stiffness provides much of the stability to plates with buckling resistance approaching yield stress. It is for this reason that the term P_{nTOR} , used to calculate the compression resistance in the proposed method, is derived from the component of elastic critical buckling due to torsion.

3.4 The relative contributions of torsion and transverse bending

In plates with buckling resistance significantly below yield, the dominant plate effect is the transverse bending of the plate between the webs. In wider plates, the effects of both transverse bending and torsion are small and may be so small as to be insignificant in wide plates. In the equilibrium equation for a flat plate under longitudinal compression, the relative contributions from torsional stiffness and transverse bending stiffness may be difficult to determine. The attraction of the torsional stiffness is that it gives a simple term to be used in the design method. Both torsional stiffness and transverse bending stiffness can be expressed in terms of $1/(b/t)^2$, so, provided the stiffness term gives a satisfactory solution, the source of the stiffness is unimportant.

4. Derivation of the terms P_{cr} and P_{crT}

4.1 Introduction

The two terms P_{crT} and P_{cr} are derived from the elastic critical buckling load for a stiffened plate. This paper considers two approaches, (a) the simplified stiffened plate approach and (b) the compression member on elastic restraint approach. The simplified stiffened plate approach is more accurate, but the compression member on elastic restraint gives a simpler model, giving easier understanding of the behaviour.

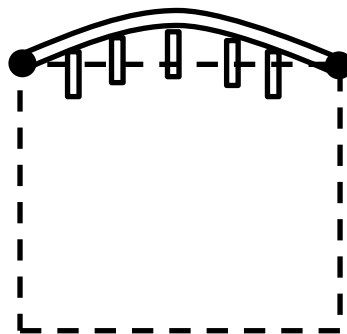
4.2 The simplified stiffened plate model

The simplified stiffened plate model assumes that the buckling mode of the plate is the same as an unstiffened plate of overall width 'b' between simply supported sides and a half-wave length of 'a'. A typical cross-section is shown in Figure 1. The mode shape is:

$$z = e \sin \frac{\pi x}{a} \sin \frac{\pi y}{b} \quad (12)$$

This excludes local buckling of the plate between the stiffeners because this effect is handled by using plate effective widths.

This model assumes the longitudinal stiffeners are of equal size and are equally spaced. It also assumes that the stiffeners add no torsional stiffness to the plate and that the stiffeners are stable.



Model for torsion and bending of plate

Figure 1: Models for a plate with multiple longitudinal stiffeners.

The behavior, excluding local buckling between stiffeners, is the sum of the longitudinal stiffener behaviour plus the unstiffened plate behaviour. The equilibrium equation is:

$$F \frac{\pi^2}{a^2} eS_a S_b = \left(EI \frac{\pi^4}{a^4} + EcI_p v \frac{\pi^4}{a^2 b^2} \right) eS_a S_b + cG \frac{t^3}{3} \frac{\pi^4}{a^2 b^2} eS_a S_b + cEI_p \left(\frac{\pi^4}{b^4} + v \frac{\pi^4}{a^2 b^2} \right) eS_a S_b \quad (13)$$

where F is the elastic critical buckling load,

n is the number of longitudinal stiffeners

I is the inertia of the longitudinal stiffener plus associated plate

I_p is the effective inertia of a unit width of plate, $I_p = \frac{t^3}{12(1-\nu^2)}$

and c is the width of plate associated with each stiffener, so $c = b/(n+1)$.

This simplifies to:

$$F = \left(EI \frac{\pi^2}{a^2} + E \frac{b}{(n+1)} I_p v \frac{\pi^2}{b^2} \right) + \frac{b}{(n+1)} G \frac{t^3}{3} \frac{\pi^2}{b^2} + \frac{b}{(n+1)} EI_p \left(\frac{\pi^2 a^2}{b^4} + v \frac{\pi^2}{b^2} \right) \quad (14)$$

This may be re-arranged as:

$$F = EI \frac{\pi^2}{a^2} + E \frac{b}{n+1} I_p v \frac{\pi^2}{b^2} + \frac{\pi^2/(n+1) G t^3}{3} \frac{1}{b} + \frac{\pi^4}{(n+1)} \frac{EI_p}{b^3} \left(1 + v \frac{b^2}{a^2} \right) \frac{a^2}{\pi^2} \quad (15)$$

In the majority of designs, the terms including Poisson's ratio have a very small effect so may be neglected, so the formula reduces to

$$F = \left(EI \frac{\pi^2}{a^2} \right) + \frac{b}{(n+1)} G \frac{t^3}{3} \frac{\pi^2}{b^2} + \frac{b}{(n+1)} EI_p \left(\frac{\pi^2 a^2}{b^4} \right) \quad (16)$$

Re-arranging the terms and writing $a = L$, this becomes

$$F = \left(EI \frac{\pi^2}{L^2} + \frac{\pi^4}{(n+1)} \frac{EI_p L^2}{b^3 \pi^2} \right) + \frac{\pi^2}{3(n+1)} \frac{G t^3}{b} = P_{cr} + P_{crT} \quad (17)$$

in which

$$EI \frac{\pi^2}{L^2} + \left[\frac{\pi^4}{(n+1)} \frac{EI_p}{b^3} \right] \frac{L^2}{\pi^2} = EI \frac{\pi^2}{L^2} + k_{PB} \frac{L^2}{\pi^2} = P_{cr} \quad (18)$$

and

$$\frac{\pi^2}{3(n+1)} \frac{G t^3}{b} = k_J \frac{G t^3}{b} = P_{ctT} \quad (19)$$

In the majority of practical cases, plates with more than two longitudinal stiffeners need transverse stiffeners to define the buckling lengths. Without transverse stiffeners, the buckling length is very long, so the design is inefficient. Where there are several longitudinal stiffeners

and transverse stiffeners, the plate effects are commonly insignificant, so they can be the same as for continuous compression members without restraint from the plate.

4.3 Simplified column on elastic restraint model for a single central stiffener

The simplified model is shown in Figure 2.

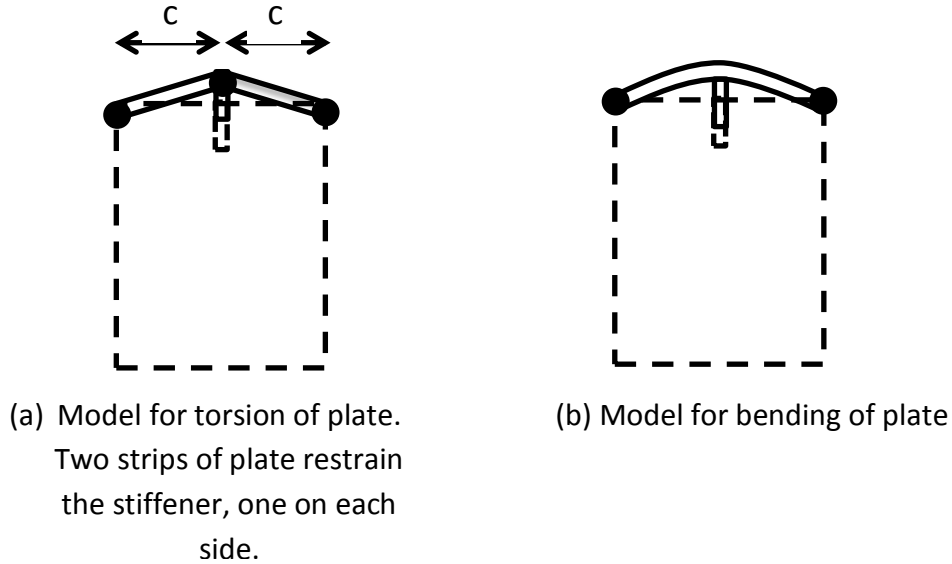


Figure 2: Models for a single longitudinal stiffener.

The increase in resistance from torsional effects can be calculated from the torsional stiffness of a strip of plate with width equal to the distance, c , from the stiffener to the adjacent web. To simplify the calculation, the strip is assumed to be pinned along both the stiffener and the web. This contrasts with the actual continuity of the plate spanning from web to web, but comparison with the simplified stiffened plate model shows the effect of the approximation is conservative. The increase in resistance from the out of plane bending of the plate spanning from web to web is calculated assuming the plate is pinned along the webs but continuous elsewhere. The equilibrium equation is

$$F \frac{\pi^2}{L^2} e \sin \frac{\pi x}{L} = EI \frac{\pi^4}{L^4} e \sin \frac{\pi x}{L} + 2GK \left(\frac{1}{c^2} \right) \frac{\pi^2}{L^2} e \sin \frac{\pi x}{L} + k_{PB} \left(1 + \nu \frac{b^2}{L^2} \right) e \sin \frac{\pi x}{L} \quad (20)$$

Removing the Poisson term and reordering, the elastic critical buckling force is given by:

$$F = \left(EI \frac{\pi^2}{L^2} + \frac{48EI_p L^2}{b^3 \pi^2} \right) + \frac{4 Gt^3}{3 b} = P_{cr} + P_{crT} \quad (21)$$

in which

$$EI \frac{\pi^2}{L^2} + \left[\frac{48EI_p}{b^3} \right] \frac{L^2}{\pi^2} = EI \frac{\pi^2}{L^2} + k_{PB} \frac{L^2}{\pi^2} = P_{cr} \quad (22)$$

and

$$\frac{4Gt^3}{3b} = k_J \frac{Gt^3}{b} = P_{crT} \quad (23)$$

4.4 Simplified column on elastic restarint model for two stiffeners

The simplified model for two stiffeners, see Figure 3, is similar to the model for one stiffener.

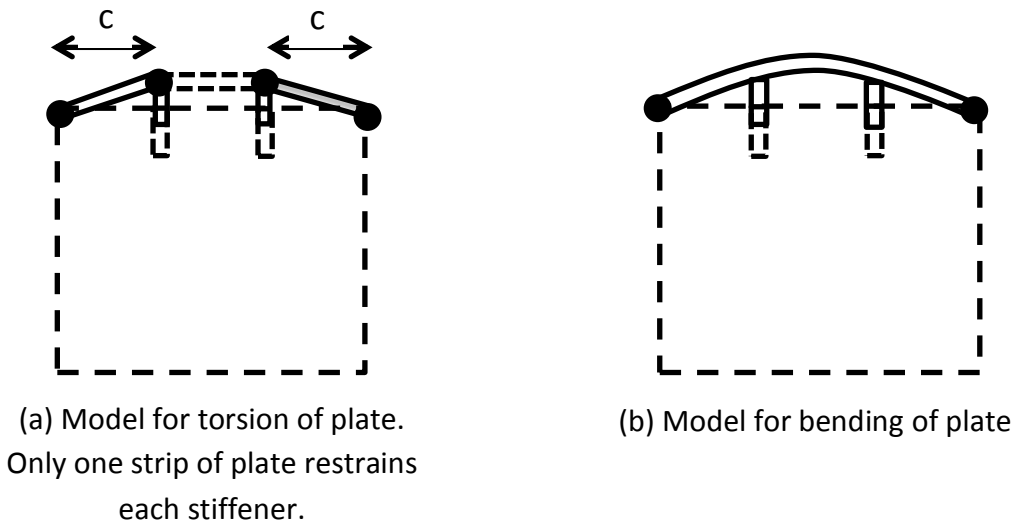


Figure 3: Models for a plate with two longitudinal stiffeners.

The elastic critical buckling force is given by:

$$F = \left(EI \frac{\pi^2}{L^2} + \frac{32.4EI_p}{b^3} \frac{L^2}{\pi^2} \right) + \frac{2Gt^3}{3b} = P_{cr} + P_{crT} \quad (24)$$

in which

$$EI \frac{\pi^2}{L^2} + \left[\frac{32.4EI_p}{b^3} \right] \frac{L^2}{\pi^2} = EI \frac{\pi^2}{L^2} + k_{PB} \frac{L^2}{\pi^2} = P_{cr} \quad (25)$$

and

$$\frac{2Gt^3}{3b} = k_J \frac{Gt^3}{b} = P_{crT} \quad (26)$$

4.5 Comparison of the plate model with beam on elastic foundation models

The plate model gives

$$F = EI \frac{\pi^2}{a^2} + E \frac{b}{n+1} I_p \nu \frac{\pi^2}{b^2} + \frac{\pi^2/(n+1) G t^3}{3 b} + \frac{\pi^4}{(n+1) b^3} \frac{EI_p}{\pi^2} \left(1 + \nu \frac{b^2}{a^2}\right) \frac{a^2}{\pi^2} \quad (27)$$

For a single stiffener, this becomes

$$F = EI \frac{\pi^2}{a^2} + E \frac{b}{n+1} I_p \nu \frac{\pi^2}{b^2} + \frac{4.93 G t^3}{3 b} + 48.7 \frac{EI_p}{b^3} \left(1 + \nu \frac{b^2}{a^2}\right) \frac{a^2}{\pi^2} \quad (28)$$

For two stiffeners, this becomes

$$F = EI \frac{\pi^2}{a^2} + E \frac{b}{n+1} I_p \nu \frac{\pi^2}{b^2} + \frac{3.29 G t^3}{3 b} + 32.5 \frac{EI_p}{b^3} \left(1 + \nu \frac{b^2}{a^2}\right) \frac{a^2}{\pi^2} \quad (29)$$

The transverse bending terms are very close to the beam on elastic foundation model if the Poisson effects are included, but the Poisson effects are commonly small in longitudinally stiffened plates because b^2/a^2 is small. The torsion term is higher in the simplified plate model because the hinge assumed in the column on elastic restraint model reduces the strain energy in the solution. The values of k_J and k_{Jf} need to be selected by calibration with tests and finite element analysis.

5. Longitudinal stiffener resistance

5.1 Introduction

The stiffener, comprising the outstand plus the half-panels of plate either side of the outstand, fails by flexural buckling restrained by the transverse bending of the plate and the torsional effects. The torsional term is removed because it is added directly into the resistance equation. With the torsional effects removed, the reduced elastic critical buckling term is:

$$P_{cr} = EI \frac{\pi^2}{L^2} + \left[\frac{\pi^4}{(n+1) b^3} \frac{EI_p}{\pi^2} \right] \frac{L^2}{\pi^2} = EI \frac{\pi^2}{L^2} + k_{PB} \frac{L^2}{\pi^2} \quad (30)$$

This force, P_{cr} , is taken as the elastic critical buckling force of the longitudinal stiffener and is used to calculate the flexural buckling resistance of the stiffener.

5.2 Effective length, L

If there are transverse stiffeners of sufficient stiffness, these will define the maximum effective length. Otherwise the effective length for minimum resistance can be found from the minimum value from the differential of the equation for the elastic critical buckling load. The effective length, or buckling length, is the length of half-sine curve for minimum resistance can be found from the minimum value from the differential of the simplified stiffened plate equation.

$$\begin{aligned} \therefore \frac{dP_{cr}}{da} &= \frac{d}{da} (n+1) EI_s \pi^2 a^{-2} + \frac{d}{da} E b I_p \left(\pi^2 a^{-2} + \nu \frac{\pi^2}{b^2} \right) + \frac{d}{da} G \frac{b t^3 \pi^2}{3 b^2} \\ &\quad + \frac{d}{da} E b I_p \left(\frac{\pi^2 a^2}{b^4} + \nu \frac{\pi^2}{b^2} \right) \end{aligned} \quad (31)$$

$$= -2(n+1) EI_s \pi^2 a^{-3} - 2E b I_p \pi^2 a^{-3} + 2E b I_p \frac{\pi^2 a}{b^4} \quad (32)$$

Maximum P_{cr} occurs at $\frac{dP_{cr}}{da} = 0$

$$\therefore -2(n+1)EI_s\pi^2a^{-3} - 2EbI_p\pi^2a^{-3} + 2EbI_p\frac{\pi^2a}{b^4} = 0 \quad (33)$$

Divide through by $2E\pi^2$ and multiply through by a^3 :

$$\therefore -(n+1)I_s - bI_p + bI_p\frac{a^4}{b^4} = 0 \quad (34)$$

$$\therefore bI_p\frac{a^4}{b^4} = (n+1)I_s + bI_p \quad (35)$$

$$\therefore a = b\left(\frac{(n+1)I_s + bI_p}{bI_p}\right)^{0.25} \quad (36)$$

Writing $a = L$, this becomes

$$L = b\left(\frac{(n+1)I_s + bI_p}{bI_p}\right)^{0.25} \quad (37)$$

The same formula can be derived from the expressions for one and two stiffeners above in terms of the stiffness/unit length for transverse bending, k_{PB} ,

$$\therefore L = \left(\frac{EI\pi^4}{k_{PB}}\right)^{0.25} \quad (38)$$

5.3 The buckling curve for longitudinal stiffeners

The flexural buckling curve for the stiffeners needs to be identified. The calculations in this paper have been made using the AASHTO buckling curve for compression members and the Eurocode curve 'c' with modified imperfection factor as specified in EN 1993-1-5 #4.5.3(5).

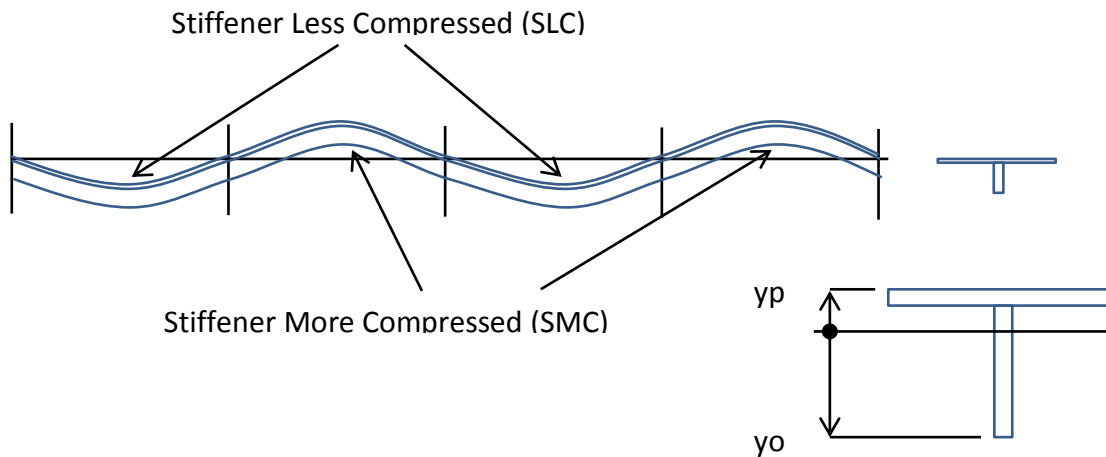


Figure 4: Continuous stiffened plate showing segments in which the stiffener is more compressed, SMC, and the stiffener is less compressed, SLC.

The buckling of the longitudinal stiffener is comparable to the buckling of a strut. There are two different deflected forms, one with the stiffener more compressed (SMC) and one with the stiffener less compressed (SLC) as shown in Figure 4. Assuming the same magnitude of initial imperfection throughout and a sinusoidal imperfection form, the strains will be highest at the tip of the stiffener outstand in the stiffener more compressed (SMC) zone because the distance from the outstand to the Neutral Axis, y_o , is greater than the distance from the plate extreme fibre to the Neutral Axis, y_p . Therefore the SMC zone begins to yield first while the SLC zone remains elastic. As the applied load is increased, the imperfection in the SMC zone would grow faster than in the SLC zone if there were no continuity because the yielding reduces the stiffness. However, the continuity means that the deflection of the SMC zone is restrained by the SLC zone. This restraint is not available to pin-ended struts. Therefore the compression resistance should be higher than for a pin-ended strut of the same section properties and with the same initial imperfection. It is possible that the Eurocode approach of increasing the imperfection factor to allow for the L/500 fabrication is more conservative than necessary to achieve the appropriate reliability.

6. Comparison with existing codes

The resistance calculated by the proposed method is compared with AASHTO LRFD and Eurocode 3 Part 1.5. For brevity, only three comparisons are made. All are for stiffened plates with a single longitudinal stiffener. Figures 5 and 6 show resistance versus plate thickness for plates without transverse stiffeners, with total plate width of 2000 mm and yield stress 345 MPa (50 ksi). Figure 5 is for longitudinal stiffener size of 400 mm \times 40 mm and Figure 6 is for size 250 mm \times 25mm. In these comparisons, the product of torsional constant and the calibration factor, $k_J \times k_{Jf}$, has been given the value 2.0; a higher value would give a longer plateau.

Figure 7 shows the resistance versus effective length is for panels with transverse stiffeners is derived from the design of the Messina Straits bridge as the design evolved. The total plate width is 4000 mm, the plate thickness is 80 mm, the stiffener size is 600 mm \times 60 mm and the yield stress in 460 MPa. It is unusual to have plates of these proportions with transverse stiffeners, but this revealed the anomaly in the Eurocode interpolation formula.

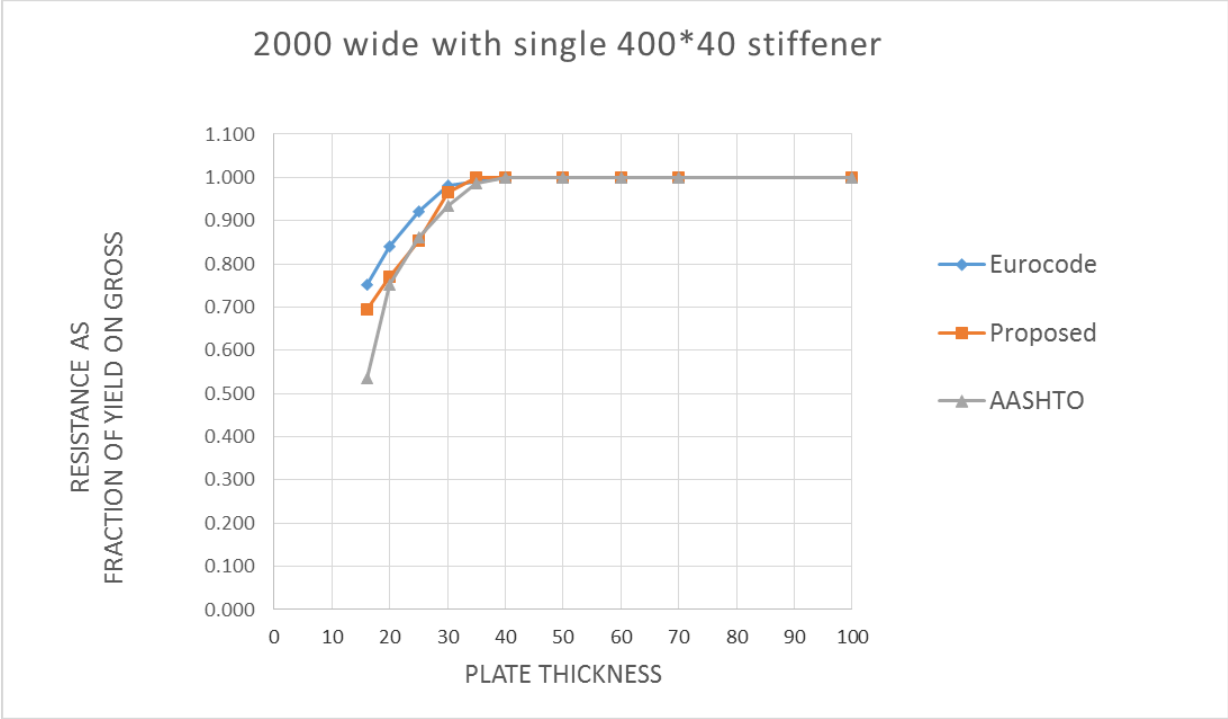


Figure 5: Comparison of codes for different plate thickness.

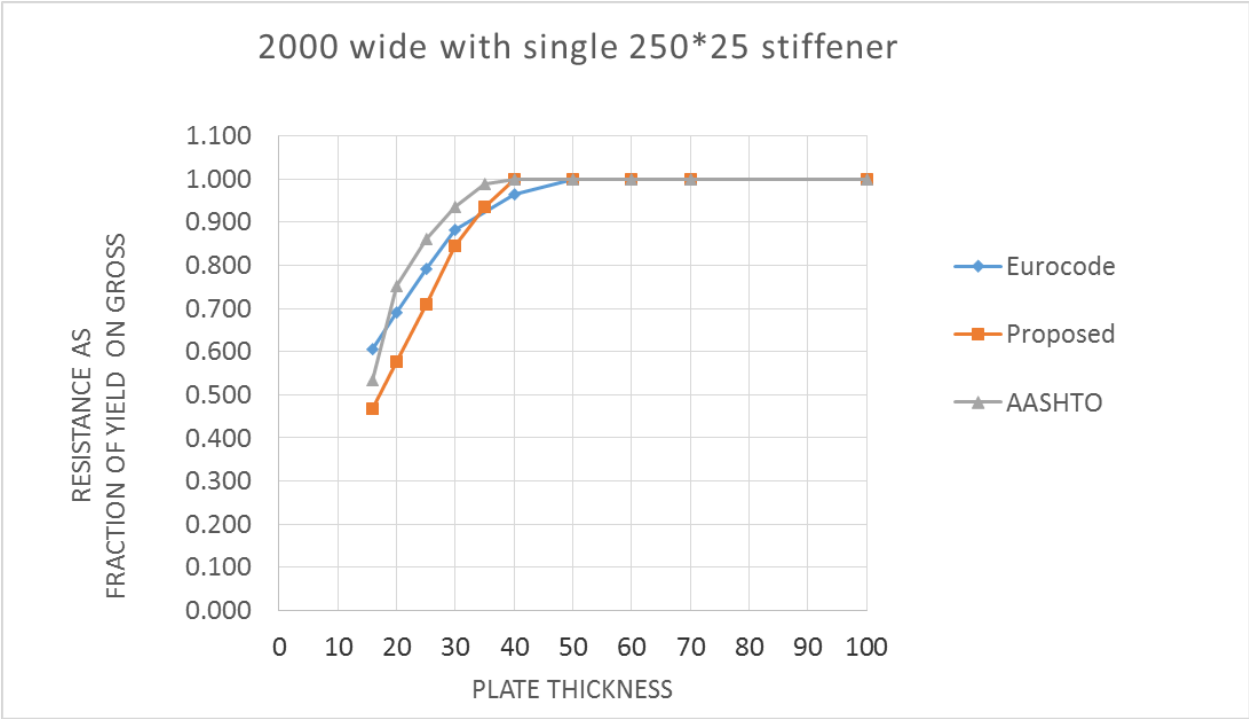


Figure 6: Comparison of codes for different plate thickness.

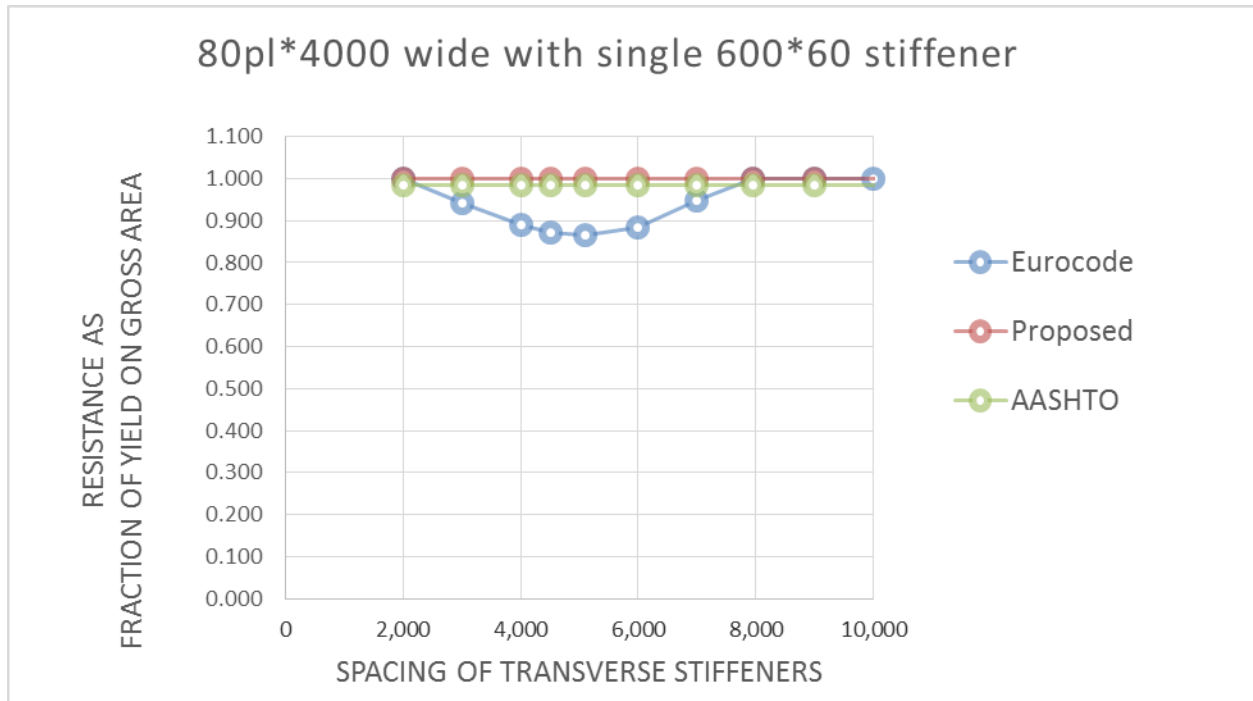


Figure 7: Comparison of codes for different buckling lengths.

7. Effect of coincident shear stresses

The destabilising effect on longitudinal stiffeners of coincident shear stresses needs to be considered. In BS 5400-3, there is an allowance that varies between zero and the same effect as longitudinal direct stresses depending on the slenderness of the panel. To agree with AASHTO's avoidance of combined bending and shear, the proportions where shear is insignificant need to be defined.

8. Design calculations – stresses or forces

Steel design codes have increasingly adopted calculation of capacity by forces and moments rather than by stresses. This is essential for exploiting the plastic and partially plastic capacity of cross-sections. Interaction formulae for different combinations of moments and forces require extensive programs of experiments and analyses to ensure they are reliable. Most steel sections are reasonably compact, so this is achievable.

Large longitudinally stiffened boxes such as those in towers and decks of big bridges are different from the majority of other steel sections.

- (1) Their ability to accept compressive plastic strains is limited to the corners, so there is no significant advantage in being able to predict the partially plastic capacity.
- (2) The range of shapes and proportions of stiffened boxes is so great that the scope of experiments and analysis to ensure the reliability of interaction formulae based on forces and moments would be enormous.
- (3) Because plasticity has little application in the resistance of stiffened boxes, stress distributions are generally elastic, so calculation capacity by stresses is convenient.

Using stresses, it is easy to calculate the resistance of a member subject to axial forces and biaxial bending.

- (4) Longitudinally stiffened boxes tend to be large and often interconnect with other members with lesser dimensions. A simple example is that of the towers of a suspension bridge in which the cross beams between the legs are commonly much narrower than the legs. The difference in dimensions means that they are many locations in a member where the capacity of local panels must be assessed rather than the capacity of whole cross-sections. It is simplest and most logical to calculate these local effects by stresses.

Therefore, codes should give the option of designing with stresses and elastic stress distributions.

9. Further research

Further research needs include finite element analyses:

- (1) to find a suitable flexural buckling curve for uniform compression,
- (2) to find a suitable value of k_{Jf} .
- (3) to establish the interaction with (a) shear and (b) in-plane bending.
- (4) to see if the effect of overall curvature of the member has a significant effect on the resistance of the stiffened plates.

Measurements of typical residual stresses in plates for different production methods and different thicknesses are needed to add confidence in finite element analyses. It is also desirable to conduct a few full scale tests to calibrate the finite element analysis.

10. Conclusions

A straightforward design approach is proposed that is simple to follow because it is based on the flexural resistance of a compression member, accounting for the transverse bending of the plate, with a simple addition for plate torsional effects. It needs only one sequence of equations for longitudinally stiffened plates either without or with transverse stiffeners. Compared with AASHTO LRFD, it allows effective widths of plate to be incorporated, which is not done in AASHTO 6.11.8. For plates with transverse stiffeners, it avoids the limitations which (a) forbid the use of the transverse stiffened rules for transverse stiffener spacing greater than three times the breadth of the plate and (b) require longitudinal stiffeners to have a certain minimum stiffness and gives no advantage for greater stiffness. Compared with the Eurocode, it is slightly simpler because it avoids the interpolation between a "plate" resistance and a "column" resistance and so avoids the anomaly in the Eurocode interpolation formula.

11. References

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