Splices in Plastically Designed Continuous Structures

WILLARD H. HART AND WILLIAM A. MILEK

PLASTIC DESIGN is based on the ductile strength of steel—the ability of structural steel to deform at and above the yield point at a uniform level of stress. If the load on a steel structure is increased sufficiently, the material will pass through the elastic range into the plastic range of stress.

At overloads, highly stressed areas are permitted to yield in bending. When such highly stressed portions of a continuous structure yield, they merely refuse to accept larger bending moments and transfer the requirement for additional moment capacity to areas which are stressed to a lesser degree. Thus, as a sufficiently large overload is applied to a continuous structure, the relative size of the positive and negative moments will be readjusted so that the structure can support additional load.

This moment redistribution does not take place at the design load. The objective is to design the structural frame so that the redistribution can take place if large overloads are imposed. If the structure is not loaded above the design load, it will act within the elastic range in a manner no different from a structure designed by elastic design procedures.

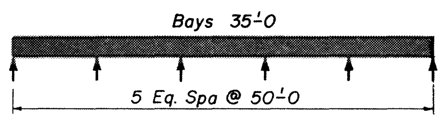
The plastic analysis for a structure may be handled either mathematically or semigraphically, by the equilibrium method or by the mechanism method. In any case, the proper solution will be one which satisfies the following three conditions: (1) equilibrium will be maintained, (2) a mechanism will develop, and (3) the plastic moment will not be exceeded any place in the structure. Thus, in a fully restrained beam there will be maximum negative moments at the two ends and a point of maximum positive moment near the center equal to the plastic moment capacity of the selected beam.

For beams restrained at one end only, such as end spans of a continuous beam, there will be only two peak moments—at one end and at the point of maximum positive moment. In either case, between the points of maximum positive and maximum negative moment there will be points of inflection. Such points are frequently selected as locations for field splices. Since the moment diagram indicates zero moment capacity required at ultimate load, many designers have misinterpreted the requirements of Part 2 of the AISC Specification and merely provide shear connections. Frequently, adequate recognition is not given to the fact that the validity of the assumption of zero moment requirement at a specific point may be negated under a distribution of loading which is different from the assumed load.

The purpose of this paper is to highlight the importance of adequate splice design to the plastic analysis of continuous beams.

LOADS LESS THAN ULTIMATE (EXAMPLE 1)

It is required to design a five-span continuous beam in which each span is to be 50 ft (Fig. 1). Bays are to be 35



Dead Load	15 psf
Live Load	30 psf
Total	45 psf
w_w = 45 x 35 =	1.6 klf

Simple Plastic Analysis

Ult. Load = 1.70 x 1.6 = 2.72 K/F

M_{EXT} = 0.0858 x 2.72 x 50^2 = 582 K.F.
24 WF 76, M_p = 600 K.F.

M_{INT} = 0.0625 x 2.72 x 50^2 = 425 K.F.
21 WF 62, M_p = 432 K.F.

Figure 1

Willard H. Hart is Regional Engineer, American Institute of Steel Construction, Milwaukee, Wisc.
William A. Milek is Research Engineer, American Institute of Steel Construction, New York, N. Y.

ft on center. The total dead load plus live load for working load is to be 45 lbs per sq ft. Thus, the beam will be subjected to a working load of 1.6 kips per lineal ft.

To determine the required beam size for such a situation by the mathematical method, it is necessary only to first multiply the working load 1.6 × 1.70, the load factor specified in Sect. 2.1 of the Specification, in order to determine that the required ultimate load capacity of the beam would be 2.72 kips per ft (w_u).

Page A6 of *Plastic Design in Steel*[1] (see chart for Equal Spans—Splice in Second Bays) indicates that the required plastic moment for the two end spans would be equal to $0.0858 w_u l^2$. For the conditions considered, the required moment capacity of the beam for the end span would be 582 kip-ft. A 24 W 76 beam of A36 steel, having a plastic modulus of 200.1, would provide a moment capacity of 600 kip-ft and might be selected for the end spans.

The moment capacity required for the interior spans would be $0.0625 w_u l^2$, which for the problem being considered would indicate a requirement of 425 kip-ft. A 21 W 62 beam would provide a moment capacity of 432 kip-ft.

Also shown on page A6 of the plastic design manual are factors which indicate the distance from the points of support to points of zero moment at ultimate load. The distance to the inflection point at the end of the cantilever in the second span would be 10.2 ft (Fig. 2). Since no bending strength would be required at this point under the assumed load, a shear splice is frequently indicated. In the interior spans, use would be made of 21 W 62 beams, again with splices indicated at points of zero moment in order to reduce the length of individual pieces to those which can be conveniently handled in the field and to facilitate the erection of the structure.

This is a rather typical solution for this type o problem. However, instances of unsatisfactory performance and even failures have occasionally occurred at loads less than the assumed ultimate, even though adequate strength was provided in the main material.

By further consideration of the action of the structure in Example 1, it will be possible to determine some reasons for unsatisfactory performance.

It was stated earlier that a *continuous* beam, when subjected to appreciable overloads, will redistribute moment and call upon the less highly stressed areas of the beam to provide the additional capacity to support the overload. It was on this assumption that the factors for the proportioning of the beam in this example were based. The reason the structure might fail to satisfactorily carry the load is to be found in the fact that by the

1. Plastic Design in Steel *American Institute of Steel Construction, New York, N. Y., 1959.*

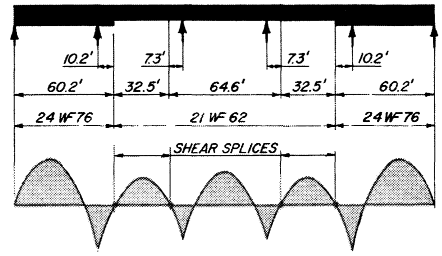


Figure 2

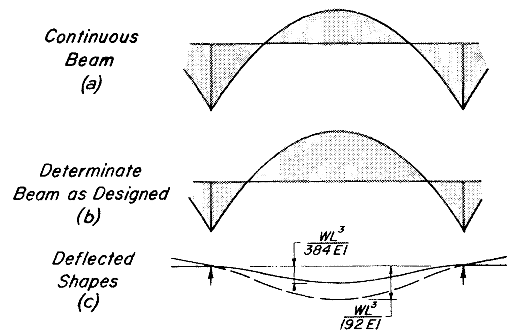


Figure 3

introduction of pure shear splices at the theoretical points of inflection at ultimate load, the beam is reduced to a simple structure consisting of simple beams supported at the ends of beams cantilevered beyond the supports of adjacent spans.

Figure 3a indicates diagrammatically the moment curve for a beam in the elastic range fully fixed at both ends. The moments at points of support are double that at midspan. However, in the example being considered, shear splices have been introduced at points of zero moment at ultimate load—midspan moment equal to end moments (see Fig. 3b). With such a situation, the moment at the supports would always be equal to the moment at midspan. At stresses in the elastic range the beam would have to rotate at points of support, rather than being fully fixed, in order for the end moments to equal the midspan moment. The midspan deflection would be double that of a truly fixed-end beam (Fig. 3c).

The deflected shape at *working load* of the beam which has been selected in this example is shown in Fig. 4. Even though subjected to an assumed uniformly distributed load, the beam would be deflected downward slightly over 2 in. in the first, third, and fifth span, but would be deflected upward in Spans 2 and 4 due to the presence of splices which are incapable of carrying moment in Spans 2 and 4 and which force the negative moments to be equal to the positive moments. Under such a condition, with an applied load of a type which was free to migrate, such as a heavy downpour of rain—

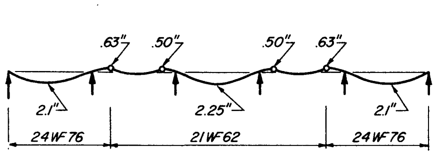


Figure 4

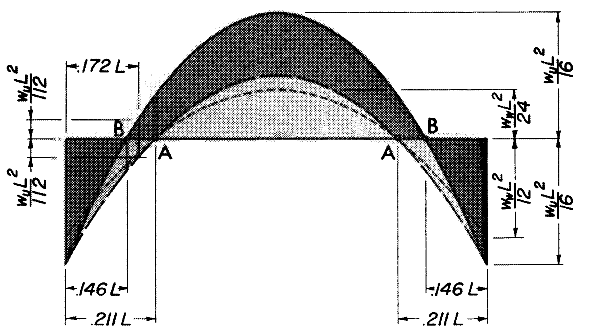


Figure 5

perhaps with a roof drain plugged or restricted—it would be impossible to apply the assumed uniform load. Water would drain from the uplifted Spans 2 and 4 and concentrate in Spans 1, 3, and 5. Spans 1, 3 and 5 then would be deflected downward to a greater amount, allowing more water to collect, while the counterbalancing loads in Spans 2 and 4 would refuse to accumulate. It can be seen that loads very different and more critical than assumed in the analysis could result and bring about failure of the structure.

It is apparent that very careful consideration must be given to the effect of the introduction of pure shear splices in a continuous beam, since under certain types of load such splices may make the assumption of uniform loading invalid.

There is one other point in connection with the members which were selected for this example. Section 2.1 of the AISC Specification says in part, "Where plastic design is used as the basis for proportioning continuous beams and structural frames, provisions relating to the allowable working stress, contained in Part 1, are waived." The intent was that the allowable unit stress provisions of Sect. 1.5, 1.6 and 1.7 would not be applicable to plastic design. On the other hand, a number of designers have interpreted Sect. 1.13, Deflections, as being a "working stress" provision, due to the fact that the depth-to-span ratio for beams in flat roofs is required to be at least equal to $f_b/600{,}000$. Even though f_b is defined as the working stress in the Specification, the intent of the Specification is that Sect. 1.13 is a deflection provision and is therefore applicable to Part 2.

For the example in question, satisfactory accuracy in checking this provision would be realized if it is assumed that the working stress would be about 24,000 psi. On the basis of this assumption, the minimum depth of beam that should be used for a 50 ft span in a flat roof is 24 in., even though the 21 WF 62 beam provides adequate plastic bending strength for the assumed loads. A 24 in. deep beam should be used in order to provide added stiffness.

Further consideration of splices in the specific case of a fixed-end beam supporting several different intensities of distributed load is shown in Fig. 5. The dotted line indicates the moment curve for such a beam at working load. The moment at points of support is twice that at midspan, points of inflection being located at **A**, 0.211 times the span length from the points of support. As overload is applied the moment curve would eventually be that shown by the dashed line, in which the extreme fiber stresses at points of support would be equal to the yield point of the material. The point of inflection remains at **A** and the moment at midspan is still equal to one-half that at points of support. If the load is increased, further yielding will commence at the points of support, and in the interest of simplification, neglecting the slight increase due to shape factor, the moment at the points of support will remain constant. The additional strength required of the beam in order to continue to support the load would have to be provided by the center portions of the span. As additional load is applied, the relative magnitude of the moments will change and the points of inflection will migrate toward the points of support, from **A** to **B**.

If a splice is located at **A**, it would need moment capacity equal to the distance from the reference line to the upper curve in order to provide for full ultimate strength. If the splice was located at **B**, it would require a moment capacity equal to the distance from the reference line to the lower curve in order to achieve fully continuous action at all levels of load. Between the two points, **A** and **B**, a splice could be located and designed for a moment capacity smaller than that required at either **A** or **B**.

Using the geometry of a parabola, it can be shown that for a *fully fixed-end beam* a splice located 0.172 times the span from the point of support with a moment capacity of $w_u l^2/112$ would be adequate for all conditions. As a simple, slightly more conservative rule, perhaps it would be well to propose that splices in *fixed-end beams* be located at the $\frac{1}{6}$ point of the span and that they be adequate to resist a moment equal to $\frac{1}{6}$ of the strength of the beam.

If the recommendation for relocation of the splice, as outlined, and the requirement that a beam not less than 24 in. deep be used on the 50 ft span, then the final recommended design for the example would appear as shown in Fig. 6. The suspended span has been moved to Span 3 and the splice point has been moved out from 7.3 ft to 8 ft-6 in. The splice in Span 2 has been moved from 10.2 ft out to 11 ft-4 in. A simple splice with adequate moment capacity could be provided by welding end plates to the beam which would then be connected by means of high strength bolts in direct tension located as close as practicable to the top and bottom flanges.

Stiffeners should be provided over the columns to prevent lateral buckling of the beam at point of high negative moment. Many other items require detailed consideration to complete the design, but their discussion would not contribute to the objective of this paper.

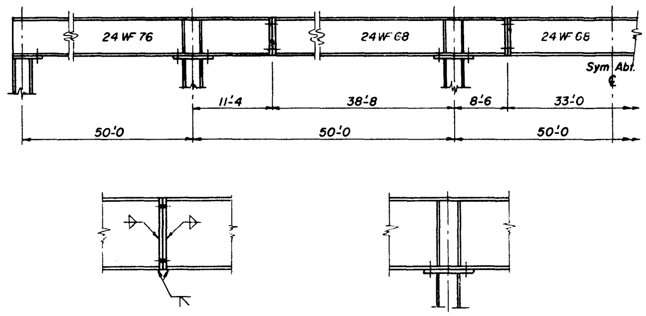


Figure 6

ALTERNATE SPAN LOADING (EXAMPLE 2)

To illustrate a slightly different aspect of a similar problem, consider the problem of designing a continuous beam on 40 ft spans to carry an ultimate load of 3 kips per ft (Fig. 7). In this case, a semigraphical analysis has been used.

The center span is fully fixed at both reactions; therefore, the required plastic moment capacity of the beam for this span would be 300 kip-ft—midspan moment equal to moments at reaction points. If A36 steel is used, an 18 WF 50 with a plastic section modulus of 100.8 would be adequate (AISC Manual page 2-8).

In the exterior span, only simple support would be provided at the exterior column; therefore, the reference line is drawn sloping upward from zero to a point such that the moment at the first interior support is equal to the positive moment in the span. The required moment capacity is found to be 412 kip-ft. A 21 WF 62 would provide adequate strength.

One point of inflection is indicated in exterior spans while interior spans each contain two. Splices might be located at these points of low moment requirement. It should be reemphasized, however, that adequate recognition must be given to the fact that the assumption of zero moment requirement at a specific point may be invalidated under a distribution of load different from the assumed load.

Section 1.3.2, Live Load, of the AISC Specification requires that "Snow load shall be considered as applied either to the entire roof area or to a portion of the roof area, and the arrangement of loads resulting in the highest stresses in the supporting member shall be used in the design." It is the intent of the Specification that realistic consideration be given to non-uniform distribution of load. It is not intended that checkerboard loading is mandatory since such loading on a roof could hardly be considered as realistic.

To illustrate the effect of non-uniform loading, consider Case 1—dead load only in Spans 2 and 4. Figure 8 shows the moment diagram which would result from the non-uniform load when applied to the 21 WF 62 (M_p = 432 kip-ft) and the 18 WF 50 (M_p = 302 kip-ft) beams previously selected on the basis of a uniform ultimate load of 3 kips per foot. For a *continuous* beam, the only effect will be that negative moments are induced throughout the second span. These moments would be less than the moment capacities provided by the beams selected and the initial selection of members would still be valid. It can be shown in this way that when continuity is maintained non-uniform loading does not result in a requirement for larger moment capacity.

On the other hand, for Case 2 (Fig. 9), in which a shear splice is introduced at point **X**, which under full load was a point of zero moment, a point of inflection will be forced under all systems of load. (For practical reasons additional splices will be required. For the purposes of this example, however, these are not considered.) A new sloping reference line must be drawn in the second span from a point representing M_p at the second interior column intersecting the simple span moment curve for dead load at the forced point of inflection 8.16 ft from the second column. Such a line extended to column **B** indicates the maximum negative moment that can occur at the first interior support. The line would be further continued from this intersection to zero at the exterior column. The maximum positive moment in the exterior span would then be 558 kip-ft, which is 146 kip-ft larger than the plastic moment capacity supplied by the 21 WF 62 beam. The remaining spans would still be adequate.

One solution would be to select a larger beam with a moment capacity of at least 558 kip-ft. Another solution to the problem in Span 1 would be to design the splice at **X** for a moment capacity of not less than 212 kip-ft. (Such an absolute minimum required moment capacity at **X** results from taking account of the full moment capacity of the 21 WF 62 (432 kip-ft) in the positive moment area of the first span. A more practical and conservative design procedure would be to determine the required splice capacity (260 kip-ft) by scaling the distance between the reference line for Case 2 to the original reference line at **X**.) It is obvious that providing for a moment splice would be more economical.

If two shear splices were provided in the second span, Case 3 (Fig. 10), then two points of inflection would be

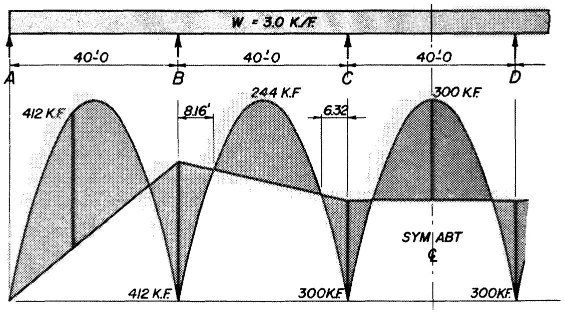


Figure 7

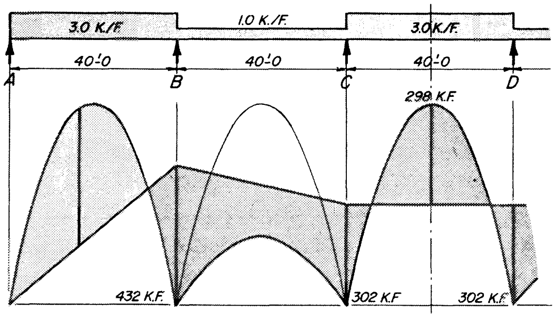
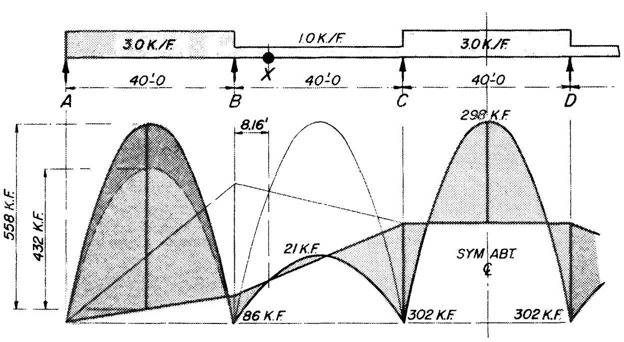
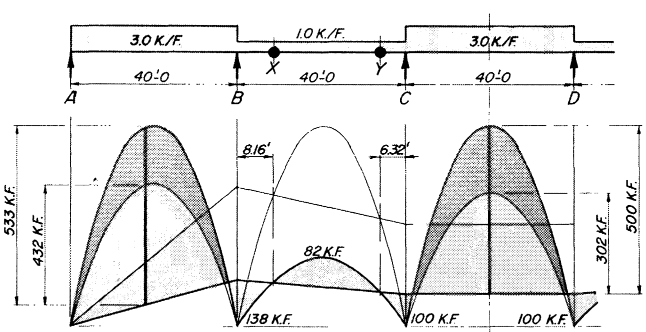


Figure 8



Required moment capacity of splice at X, 212 k.f.

Figure 9



Required moment capacity of splice at X, 212 k.f.
Required moment capacity of splice at Y, 200 k.f.

Figure 10

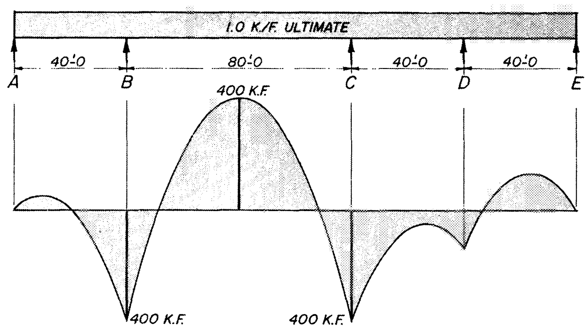


Figure 11

forced in the curve for moments under a 1 kip per ft load. The moment at the first interior support, by projecting the reference line, would be 138 kip-ft. At the second interior support, the negative moment would be limited to 100 kip-ft. Under the limitation imposed by these small end moments, the first and third spans would be deficient by an amount represented by the area between the upper and lower parabolas. In order for the selected beams to be adequate, the splice at **X** would have to be designed for not less than 212 kip-ft, and at **Y** a moment capacity of 200 kip-ft would be required. (More readily determined values would be 260 kip-ft at **X** and 212 kip-ft. at **Y**.)

VARYING SPANS (EXAMPLE 3)

With one long span in a series of continuous beams (see Fig. 11), the coefficients given in *Plastic Design in Steel* for location of the splices cannot be used. However, the semigraphical method provides a means for ready solution. Discussion of such methods can be found in the above-mentioned manual under the chapter entitled, "Design of Continuous Beams."

The effect of varying span lengths added to the effect of pure shear splices on the deflected shape in the elastic range of stress, or the effect of non-uniform loading, compounds the need for careful consideration.

SUMMARY

The true significance of shear splices (real hinges), as they affect the performance of a structure under realistic loads other than those assumed in the initial design calculations, must be considered. The coefficients which are provided in the Appendix to *Plastic Design in Steel* are rigorously correct for *continuous* beams and may be used with confidence in the design, but in the presence of real hinges at splices a given beam may be rendered discontinuous by the application of a system of loads different from the system upon which the analysis is based.

The semigraphical method provides a simple yet powerful tool which presents to the designer a graphic representation in which the effect of various loads, varying span lengths, or the strength requirement of splices is easily recognizable.